A GENERAL SCALING LAW FOR STRENGTH-DOMINATED COLLISIONS OF ROCKY ASTEROIDS.

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The empirical observation that large samples of rock are weaker than small ones first led to the suggestion that collisional breakup is scale-dependent [1]. Similarly, the observed strengthening of rock at high loading rates has been used in scaling analysis [2] and numerical modeling [3-5] to argue for scale-dependent collisional outcomes. All of these models predict that large rocky bodies should be relatively weaker than small ones in strength-dominated collisions, i.e. for rocky asteroids smaller than ~ 1 km diameter. Although these models all predict the same general trend, the magnitude of the effect varies considerably, with a factor of ~100 uncertainty in the energy per unit target mass required to shatter a 1 km asteroid.

The predicted weakening of bodies with increasing size is based on the idea that collisional fragmentation is due to the growth and coalescence of pre-existing flaws in the target. Small flaws are relatively numerous and, therefore, closely spaced, but require large tensile stresses to activate their growth. Large flaws are more widely spaced and activate at low stresses. In a small-scale collision, the tensile pulses are relatively short in duration. Therefore, only the small (closely spaced) flaws have time to coalesce. On the other hand, the tensile stress is maintained much longer in a large-scale collision, so that larger flaws have time to coalesce. Hence, the target in a small-scale collision is effectively stronger than a large-scale counterpart, because the small flaws require comparatively higher stresses for flaw coalescence, i.e. failure.

The rate at which targets weaken with increasing size can be related to the size distribution of pre-existing flaws. In particular, if

\[ N_s = k_s s^{\phi/2} \quad (1) \]

where \( N_s \) is the number of flaws of length \( s \) or greater per unit volume and \( \phi \) is the Weibull exponent, then the kinetic energy per unit target mass required for catastrophic fragmentation is

\[ Q_s^* \propto D^{-(1+3/2\phi)} \quad (2) \]

where \( D \) is the target diameter [6]. Therefore the exponent on target diameter in Eq. (2) is determined by the flaw size distribution exponent \( \phi \).

Housen and Holsapple [6] described collision tests using granite targets with a factor of 18 variation in size scale. They showed that targets do in fact weaken with increasing size, i.e. \( Q_s^* \) decreases with increasing target size, viz., \( Q_s^* \propto D^{-0.4(0.02)} \). They also measured the \textit{in situ} flaw distribution for the granite and found \( \phi=9 \), which, from Eq. (2) gives \( Q_s^* \propto D^{-0.4} \), in agreement with the collision results. This illustrates the validity of Eq. (2) and implies that an estimate of the flaw distribution appropriate for rocky asteroids could be used to determine the dependence of \( Q_s^* \) on \( D \).

Although flaw distributions for rocky asteroids are currently unknown, there is considerable information for terrestrial rocks. Figure 1 shows measurements of flaw and fault trace lengths\(^1\) over a range of 11 orders of magnitude. Each data set in Figure 1 exhibits a relatively flat slope at the smallest sizes measured and a steeper slope at the large-size end. The former is the result of an inability to identify all traces at or near the resolution limit of a given study\(^2\) while the latter is due to data “censoring,” in which long faults crossing completely through an imaged area must be ignored because their actual length cannot be determined. Noting this, a single line describes the entirety of the data remarkably well. It can be shown [6, 7] that the Weibull exponent \( \phi \) is related to the slope, \( \beta \), of a line on Figure 1 by \( \phi=2(1-\beta) \).

The straight line in Figure 1 was constructed for an idealized case in which the mean spacing between flaws of a given length is equal to the flaw length. This case is referred to as “fully cracked” because, on average, flaws of each size are sufficiently numerous to just touch each other. As shown in [6], this condition requires that \( \beta=2 \) (i.e. \( \phi=6 \)) and gives a specific value for the y-intercept of the line. It is also interesting to note that this is the only slope for which the flaw distribution is scale invariant, e.g. the size of the largest flaw in a body scales with the size of the body. A value of \( \phi=6 \) gives a much stronger dependence of \( Q_s^* \) on \( D \) than that for the granite collision tests reported in [6]. In particular, \( \phi=6 \) gives \( Q_s^* \propto D^{-2/3} \) (see Figure 2). This is not surprising in the sense that the granite specimens used in [6] were obtained from a monument supplier, who undoubtedly selected them for their lack of large flaws. Hence, the flaw distribution for these pristine samples would be steeper than that for large-scale samples of rock in the field.

The fact that the line in Figure 1 fits the trace distributions for four different rock types over a range of 8 orders of magnitude suggests that the fully cracked case may be a general description of large-scale samples of well-cracked rock. It is reasonable to expect that cooling of differentiated rocky asteroids, along with repeated subcatastrophic impacts might result in well-cracked interiors. In this case, the line shown in Figure 2 would be representative of these objects, and the weakening of rocky bodies with increasing size would be more pronounced than indicated by earlier estimates.

\(^1\) Trace length is the expression of a three dimensional flaw on a two dimensional surface of observation.

\(^2\) The counts for traces shorter than a few tens of microns are believed to be complete, so the flattening there probably reflects an actual change in the flaw distribution.

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**Figure. 1.** Distribution of measured flaw trace length for four rock types.

**Figure. 2.** Comparison of estimates of the kinetic energy per unit target mass required for catastrophic fragmentation of rocky asteroids (strength-regime only).